CENCO Physics — Selective Experiments In Physics

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# NEWTON'S LAW OF COOLING

**OBJECT**: To make a study of Newton's law of cooling, and to determine the temperature lag of a cooling body, such as a calorimeter or a thermometer.

**METHOD**: Two similar calorimeters, one blackened and one polished, are filled with warm water and placed in surroundings of known temperature. Observations of the calorimeter temperatures are made at regular intervals of time, and temperature-time curves are plotted. A semi-logarithmic graph of the data is plotted with time represented on the uniform scale and difference in temperature between the calorimeter and the room on the logarithmic scale. The semi-logarithmic curve indicates the probable maximum temperature difference over which Newton's law of cooling may be said to hold satisfactorily. From the slope of the graph, the lag of the calorimeter is determined.

**THEORY**: Newton's law of cooling is an empirical law which states that the rate of change of temperature of a body is directly proportional to the difference in temperature between the body and its surroundings, provided the temperature difference is small. The law may be shown to be consistent with a more general law of heat transfer known as the Stefan-Boltzmann radiation law, according to which the amount of energy radiated by a body per unit time is directly proportional to the fourth power of the absolute temperature of the body. Representing the time rate of radiation of energy by $R$, the mathematical statement of the law is

$$R = kAT^4 \tag{1}$$

where $T$ is the temperature on the absolute scale (˚K), A the area of radiating surface, and $k$ a constant called the "radiation constant" of the surface.

The value of the constant $k$ depends upon the character of the radiating surface. Bodies possess three related surface characteristics which are important in the transfer of heat by radiation. The *reflectivity* of a surface is the fraction of the incident energy that is reflected. The *absorptivity* of a surface is the fraction of the incident energy that is absorbed. Clearly, a body that is a good reflector is a poor absorber, and vice versa. A body that absorbs all of the radiant energy falling upon it, reflecting none, is called a "black" body. The *emissivity*, or *emitting power*, of a surface is the amount of energy emitted by it per unit area per unit time. This, according to the Stefan-Boltzmann law, varies with the temperature, and depends upon the radiation constant $k$ of the surface. Kirchhoff made the important discovery that the ratio of the emissivity to the absorptivity is the same for all bodies at any given temperature. This relationship is known as Kirchhoff's law of radiation. Thus, a body which is a good absorber (and hence a poor reflector) is also a good emitter, a black body having the maximum possible emissivity at any given temperature. It is evident that the radiation constant $k$ of a highly polished metal surface is much less than that of a matte surface of low reflectivity. The value of $k$ may be expressed in various units, e.g., $cal/cm^2/sec/°K^4$, $ergs/cm^2/sec/°K^4$, or $watts/cm^2/°K^4$. The accepted value of the black body radiation constant is $1.36 \times 10^{-12}$ $cal/cm^2/sec/°K^4$.

The radiation law as first announced by Stefan in 1879 was based upon experimental data. Boltzmann showed from theoretical considerations that the law holds strictly only for a black body. It is found to hold very closely, however, for all ordinary cases of radiation.

If a body at a temperature $T$ is placed in a room the temperature of which is $T_R$, in accordance with the Stefan-Boltzmann law the net rate of exchange of radiant energy between the body and its surroundings is

$$R = kA\left(T^4 - T_R^4\right) \tag{2}$$

Equation (2) may be written

$$R = kA\left(T - T_R\right)\left(T^3 + T^2T_R + TT_R^2 + T_R^3\right) \tag{3}$$

If the difference between $T$ and $T_R$ is small in comparison with $T$ and $T_R$, Eq. (3) reduces to the approximation

$$R = 4kAT_R^3\left(T - T_R\right) \tag{4}$$

Since $k$ and $A$ are constants, if $T_R$ remains constant during the heat exchange,

$$R = K\left(T - T_R\right) \tag{5}$$

where the constant $K = 4kAT_R^3$. Equation (5) indicates that the rate of heat exchange by radiation is approximately proportional to ($T$ - $T_R$), provided this difference is sufficiently small. If $T > T_R$, $R$ is positive, indicating that the body is emitting more radiation than it receives; if $T < T_R$, the resulting negative sign of $R$ represents the fact that the body is absorbing heat from the room. Although the foregoing treatment is based upon the Stefan-Boltzmann law, and hence considers only energy exchanges due to radiation, in practice a similar relationship is found to hold when exchanges due to conduction and convection are included, *provided the effects of conduction and convection are relatively small.* In this case, of course, the constant K contains conduction and convection factors as well as the radiation constant *k*.

The time rate of gain or loss of heat which a body undergoes is directly proportional to its time rate of change of temperature *dT /dt* and depends upon its thermal capacity $c \cdot m$ according to the equation

$$\pm R = \mp c \cdot m \frac{dT}{dt} \quad (6)$$

where *c* is the specific heat and m the mass of the body. Since a positive value of *R* corresponds to a loss of heat by the body and a consequent fall in temperature, and since, furthermore, a decreasing temperature is indicated by a negative value of *dT /dt*, the signs of the two sides of Eq. (6) are always opposite. Then from Eqs. (5) and (6),

$$K(T - T_R) = -c \cdot m \frac{dT}{dt} \quad (7)$$

or

$$T - T_R = -L \frac{dT}{dt} \quad (8)$$

where $L = c \cdot m / K$. The constant *L* will later be seen to have a special significance. Equation (8) is a mathematical statement of Newton's law of cooling. It states that for a given temperature of the surroundings, the rate of temperature change of a body is (a) directly proportional to the temperature difference ($T$ - $T_R$) between the body and its surroundings, (b) directly proportional to the radiation constant k of the surface, and (c) inversely proportional to the thermal capacity $c \cdot m$ of the body.

Separation of the variables in Eq. (8) gives

$$\frac{dT}{T - T_R} = \frac{-dt}{L} \quad (9)$$

Integration of Eq. (9) yields

$$\log_e(T - T_R) = \frac{-t}{L} + C \quad (10)$$

or

$$T - T_R = e^{(-t/L)+C} = e^c \cdot e^{-t/L} = C_1 e^{-t/L} \quad (11)$$

When $t$ = 0, $T$ = $T_o$ where $T_o$ is the initial temperature of the body. Hence the constant $C_1 = T_o - T_R$ and Eq. (11) may be written

$$T - T_R = (T_o - T_R) e^{-t/L} \quad (12)$$

Representing the initial temperature difference $T_o$ - $T_R$ between the body and its surroundings by $D_o$ and the difference $T$ - $T_R$ at any subsequent time by $D$, Eq. (12) becomes

$$D = D_o e^{-t/L} \quad (13)$$

Equation (13) shows that the temperature difference between a body and its surroundings is an exponential function of the time. At a certain time when $t$ = $L$, $e^{-t/L} = e^{-1} = \frac{1}{e}$ and Eq. (13) becomes

$$D = \frac{1}{e} D_o \text{ (when } t = L) \quad (14)$$

This particular value of *t* is called the "lag" of the system, which is defined as the time required for the temperature difference to become one *e*th (about 36.8%) of the initial difference.

A method of determining the lag of a system, such as a calorimeter or a thermometer, can be obtained from an analysis of Eq. (13). Taking the common logarithm of both sides yields

$$\log_{10} D = \log_{10} D_o - \frac{t}{L} \log_{10} e \quad (15)$$

Since *e* = 2.718 and $\log_{10} e$ = 0.434, Eq. (15) becomes

$$\log_{10} D = \log_{10} D_o - \frac{0.434t}{L} \quad (16)$$

Solving for t,

$$t = \frac{L}{0.434} \log_{10} D_o - \frac{L}{0.434} \log_{10} D \quad (17)$$

or

$$t = A - B \log_{10} D \quad (18)$$

where *A* and *B* are constants, viz.,

$$A = \frac{L}{0.434} \log_{10} D_o \quad \text{and} \quad B = \frac{L}{0.434} \quad (19)$$

A semi-logarithmic graph of Eq. (18) yields a straight line of slope *B*. From the slope of an experimental graph of *t* versus $\log_{10} D$, the lag *L* can be determined. Temperature lag is an important concept, particularly in the measurement of temperature where there is a lag between the temperature of the instrument and that of the body whose temperature is being measured. The lag of a thermometer depends upon its insulation and upon the nature of the contact between the thermometer and its environment. For example, in the case of the thermoelectric thermometers used on automobiles to indicate the motor temperature, the lag depends upon the conductivity of the cylinder walls and the location of the thermocouple in the engine block. An interesting application of the problem also arises in the field of medicine in connection with recording rectal thermometers. The lag of a thermometer can be measured by the method described above. For many practical purposes the value of e may be taken as approximately equal to 3. Therefore, at the end of time intervals $t_1$ = $L$, $t_2$ = $2L$, $t_3$ = $3L$, etc., the differences between the desired temperature and the thermometer reading are approximately $D_0/3$, $D_0/9$, $D_0/27$, etc. Thus, when the initial temperature difference $D_o$ between the thermometer and its surroundings is small, any thermometer will assume (practically) the temperature of its surroundings after only a few time intervals of the value *L*. The actual time required will, of course, vary with different thermometers depending upon the value of the lag *L*, hence the importance of this constant in thermometry.

**APPARATUS**: The apparatus, represented diagrammatically in Fig. 1 and illustrated in Fig. 2, is essentially a double radiation calorimeter in which a polished tubular calorimeter cup A and a blackened one Bare suspended side by side from two holes in the cover of a vessel V. Surrounding the calorimeter cups and attached to the asbestos board cover S by threaded flanges are two cylindrical shields C and D. In use, the outer vessel is filled with water at room temperature. The calorimeter cups are thus insulated by a "dead air' space and by a water jacket. The temperatures of the liquids in the calorimeter cups are indicated by thermometers $T_A$ and $T_B$ inserted through rubber stoppers in the calorimeter cups. These thermometers, and one in the water jacket, should be graduated in tenths of a degree from 0 to 50° C. Accessory apparatus consists of a stopwatch or a clock with a sweep seconds hand, a Bunsen burner, a 500mL beaker, towels, Cartesian coordinate paper and semi-logarithmic paper.

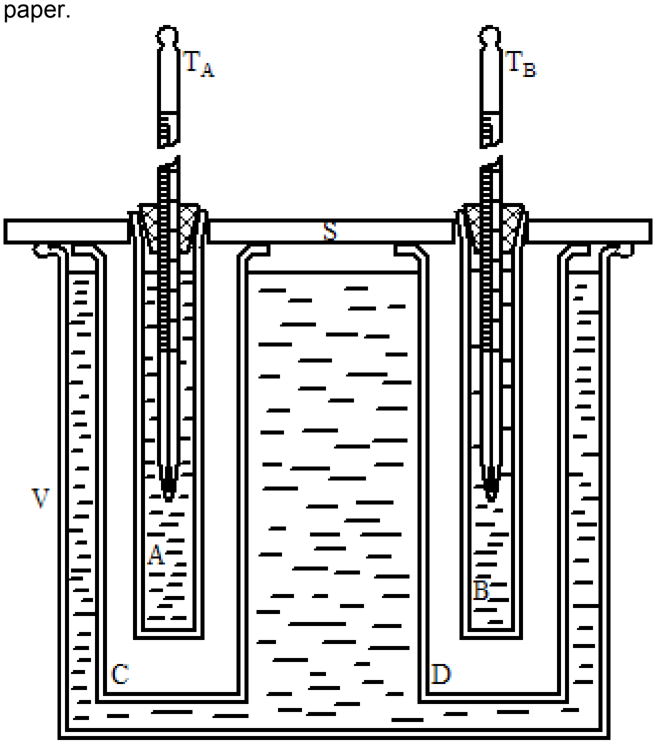

Fig. 1. Diagram of Radiation Calorimeter

**PROCEDURE**:
**Experimental**: Locate the apparatus in apart of the laboratory that is free from drafts and not near radiators. With the cover S and the cylindrical shields C and D in place, fill the outer vessel V with water at room temperature. (Note: It is a good plan to keep water standing in the water jacket at all times instead of refilling it each time the experiment is performed; this precaution makes it certain that the temperature of the water jacket is the same as that of the room.) Support a thermometer in the water jacket in such away that it can be read conveniently and quickly. The reading of this thermometer gives the room temperature (in degrees centigrade). The use of the water jacket benefits the experiment in two ways: (a) it makes it possible, by fixing the temperature of the surroundings, to determine room


Fig. 2. Radiation Calorimeter

temperature definitely, and (b) by virtue of the high thermal capacity of water, it keeps the surrounding temperature nearly constant during the experiment.
Fill the calorimeter cups A and B to within about 2cm of the top with water at about 50°C. Place them in position and insert the stoppers and thermometers. Read and record the centigrade temperatures of the cups A and B at intervals of one minute until the temperature of each has fallen to within 10° of room temperature. If a student is working alone, one thermometer should be read on the minute and the other on the half minute. When two students are working together, it is convenient to place the apparatus in the center of the table with the observers on opposite sides. Each observer then reads one of the thermometers. Since the polished calorimeter cools more slowly than the blackened one, observations on the former must be continued longer than on the latter. The temperature of the water jacket should be recorded at least every two minutes. Record the data as shown in Table I.

**Analysis of Data**:
**Required Analysis**: 1. On the same sheet of Cartesian coordinate paper, plot the data for both calorimeter cups with temperature as the ordinate and time as the abscissa (columns 3 and 4 versus column 1).

2. Compute the temperature difference $D_A = T_A - T_R$ for the blackened cup A and enter in column 5 of the table. Plot a semi-logarithmic curve of the data which appears in columns land 5 with time units indicated on the uniform scale and temperature differences on the logarithmic scale. (A discussion of logarithmic and semi-logarithmic graphs is given in the General Instructions sheet on *Graphs*.) Determine the slope of the portion of the curve corresponding to the latter part of the run. From the slope compute the lag *L*. From the curve determine the time *t* corresponding to the temperature difference $D_o/e$ and compare with the value of *L*.

**Optional Analysis**: 1. Repeat the analysis of Part 2 above for the cup B and compare the values of the lag *L* for the blackened and polished cups.

2. The differences obtained by subtracting each reading in columns 3 and 4 from the reading immediately following it give the temperature changes that occurred in the corresponding interval of time, i.e., in one minute. These differences are, therefore, the rates of cooling $\Delta T/\Delta t$ in degrees per *minute at the middle of the corresponding time intervals*. Plot a curve of the rate of cooling versus the temperature for each of the calorimeters. Discuss the physical significance of these curves. Explain the meaning of the intercepts produced by the intersection of the curve (extended) with the coordinates axes. In what way are these curves related to the curves plotted in Part 1 of *Required Analysis?*

3. Determine the mass $m_A$ of the calorimeter cup A and the mass $m_w$ of the water that it contains. From the known values of the specific heats of water and copper, calculate the thermal capacity of the calorimeter and contents. From this and the average rate of cooling over the first ten minutes, compute the heat lost by the cup A in that time.

**QUESTIONS**: 1. What do the cooling curves for the two calorimeter cups show about the respective radiation constants? Explain how these curves are consistent with Kirchhoff's law.

2. What does the semi-logarithmic graph of *t* vs. *D* show about the temperature range over which Newton's law of cooling is a satisfactory approximation?

3. Compute the maximum percentage error that would be introduced in this experiment if the energy *R* radiated per second were computed by Eq. (4) instead of by Eq. (2).

4. Explain how the lag of a calorimeter can be obtained from the cooling curves of Part 1 of *Required Analysis* when the initial temperature and the room temperature are known. Determine the values of *L* in this way and compare with the other determinations.

5. The lag of a certain thermometer is 30 seconds. If the initial temperature difference between the thermometer and its surroundings is 10°C, how long will it take for the thermometer to indicate the correct temperature within 0.1°C?

6. What is the lag of a clinical thermometer if it must be left in the mouth of a normal patient for 3 minutes in order to register to 0.2° F, assuming a room temperature of 70°F?

TABLE I

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Time (min) | Room Temperature (°C) | Temperature of A (°C) | Temperature of B (°C) | $D_A = T_A - T_B$ (°C) |
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| etc. | | | | |