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ROTAIONAL INERTIA; ANGULAR MOTION
(Electronic Timer)

OBJECT: To study angular motion and the concept of rotational inertia; in particular, to determine the effect of a constant torque upon a disk free to rotate, to measure the resulting constant angular acceleration, and to determine the rotational inertia of the disk.

METHOD: A constant torque is applied to a metal disk which is free to rotate. The disk is found to move with uniform angular acceleration. This acceleration is measured by a spark-recording method in which sparks at regular and measurable time intervals make their traces upon a coated paper, containing a polar-coordinate scale, fastened to the side of the disk. The angular acceleration is determined from the slope of the curve of angular speed plotted as a function of time. The angular speeds are calculated from the observed angular distances between spark traces and the time between sparks. The torque is the product of the measurable force applied to the edge of the disk and the radius of the disk. The rotational inertia is then determined from the ratio of the torque to the angular acceleration. Finally this "observed" value of the rotational inertia is compared with the "theoretical" value calculated from the geometrical constants and the mass of the disk.

THEORY:

Angular Speed: The angular speed of a body is defined as its time rate of change of angular distance. The average angular speed is the ratio of the angular distance which the body has traversed to the time required to travel that distance. The defining equation for average angular speed $\bar{\omega}$ is

$$\bar{\omega} = \theta / t \qquad (1)$$

where θ is the angular distance traversed and *t* is the time required for the body to travel that distance. *Instantaneous* angular speed ω, is the limit of the above ratio as the time is made vanishingly small. In symbols

$$\omega = L_{im\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad (2)$$

where $\Delta\theta$ is the small increment of angular distance traversed in the corresponding element of time Δt. The absolute units of angle and time in both metric and British systems are the radian and second, respectively, and hence the absolute unit of angular speed is the *radian per second*.

There is a distinction between angular *speed* and angular *velocity* which is similar to that between linear speed and linear velocity. The *direction* of an angular velocity is specified as the direction of its axis of spin; the *sense* of the direction is related to the sense of the rotation as the direction of advance of an ordinary right-handed screw is related to its direction of rotation. Hence the vector that represents an angular velocity is drawn in a direction parallel to the axis of spin. The vector arrow points in the direction of the thumb of a right hand which grasps the axis, with the fingers encircling the axis in the direction of the rotation.

Angular Acceleration: Whenever a body has its angular velocity changed it has an angular acceleration. This angular acceleration is defined as the time rate of change of angular velocity. Average angular acceleration is the ratio of the change in angular velocity to the time required to produce the change. In symbols, average angular acceleration $\bar{\alpha}$ is defined by the equation

$$\bar{\alpha} = \frac{\omega_t - \omega_o}{t} \qquad (3)$$

where ω_t is the final angular velocity, ω_o is the initial angular velocity and *t* is the time required to change the velocity. The instantaneous value of the angular acceleration α is the limit of this ratio as the time is made vanishingly small. Symbolically

$$\alpha = L_{im\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{dw}{dt} \qquad (3a)$$

where $\Delta\omega$, is the change in angular velocity taking place in the small increment of time Δt. The absolute British and cgs units of angular acceleration are the same, namely the radian per second^2.

If a curve of angular speed is plotted against time, as in Fig. 1, a straight line is obtained when the angular acceleration is constant. i.e., when the angular velocity changes uniformly. It is seen from Eq. (3a) that the angular acceleration is the slope of such a curve; this fact will be utilized in the present experiment.

Rotational Inertia: *Rotational inertia (also called moment of inertia) is that property of a body which causes it to oppose any tendency to change its state of rest or uniform angular velocity. It will be observed that this property is analogous in*

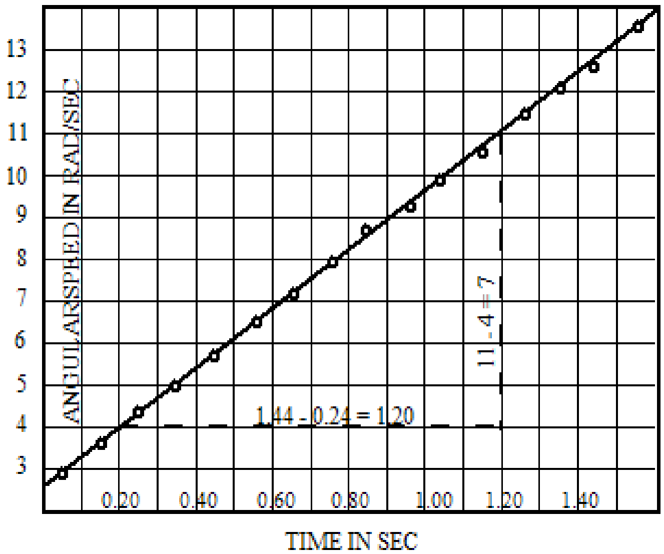

Fig. 1. Angular speed as a function of time for disk rotating under influence of a constant torque. The angular acceleration is the slope of curve $\alpha = \frac{(11.0 - 4.0)\,rad\,sec}{(1.20 - 0.20)\,sec} = 7.0\,\frac{rad}{sec^2}$

rotary motion to inertia in linear motion, as inertia is that property of matter by virtue of which the body opposes any tendency to change its state of rest or uniform linear velocity. The measure of the rotational inertia *I* of a body is the ratio of the torque *L* acting upon it to the angular acceleration a produced by that torque. In symbols the defining equation is

$$I = L/\alpha \quad (4)$$

When Eq. (4) is written in the form

$$L = I\alpha \quad (4a)$$

its analogy to the familiar equation for linear motion involving force, inertia and acceleration- namely, $f = ma$ -is apparent. It is apparent that Eq. (4a) is merely another form for stating Newtons second law of motion, as applied to rotary acceleration.

It may be shown that the value of *I* for a small particle of mass in rotation at a distance *r* from some axis of rotation is given by $I = mr^2$. For an extended body the total moment of inertia is given by the summation

$$I = \sum mr^2 \quad (4b)$$

The unit of *I* is made up as a composite unit. In the cgs system the unit of I is the gram-centimeter^2. The mks unit is the kilogram-meter^2. In the British gravitational system we obtain the mass in slugs by dividing the weight in pounds by the acceleration due to gravity (32 ft/sec^2) and hence the unit for *I* is the slug-foot^2.

The Determination of Rotational Inertia: The rotational inertia of a simple geometrical solid can be calculated by special equations derived by the use of integral calculus. For example, the rotational inertia of a uniform cylinder of radius *r* and mass M rotating about its longitudinal axis is given by

$$I = \tfrac{1}{2} Mr^2 \quad (5)$$

The rotational inertia of an object about any axis may be obtained experimentally, no matter how irregular or non-homogeneous the body may be, by applying a known torque to the body and measuring the resulting angular acceleration. From Eq. (4),

$$I = L/\alpha = fr/\alpha \quad (6)$$

where *I* is the applied force, and *r* is its moment arm.

In this experiment the rotational inertia of a circular disk is measured by applying a known torque (due to a known mass fastened to a cord wrapped around the rim of the disk), and measuring the resulting angular acceleration by the use of a technique described later.

The Accelerating Force: It might be thought at first glance that the accelerating force *I* (Fig. 2) on the disk is equal to the weight mg of the object attached to the cord. This is true

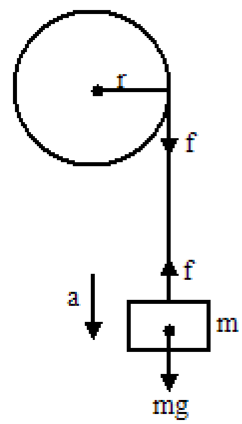

Fig. 2. The forces acting on the mass m that result in its acceleration a.

when the disk is at rest. But when the object is moving downward with an acceleration *a*, the net downward force that tends to accelerate *m* is less than *mg*. This accelerating force is *mg - f* and hence

$$mg - f = ma$$

The tension *f* in the cord is therefore given by

$$F = m\,(g-a) \quad (7)$$

The linear acceleration of the descending object is the same as that of the rim of the disk to which the cord is attached. Since the linear acceleration of a particle equals its angular acceleration multiplied by its radius of rotation, or

$$a = \alpha r \quad (8)$$

Eq. (7) may be written as

$$f = m(g - \alpha r) \tag{9}$$

The torque L_1 produced by *l* is given by

$$L_1 = Fr = m(g - \alpha r)r \tag{10}$$

Because of frictional torque *L*, the *net* torque *L* is reduced and is given by $L = L_1 - L_f$. Hence the expression for the rotational inertia Eq. (6) becomes

$$I = \frac{L}{\alpha} = \frac{m(g - \alpha r)r - L_f}{\alpha} \tag{11}$$

This is the working equation of this experiment since its use makes it possible to determine *I* in terms of the known value of *g* and measurable quantities.

APPARATUS: The chief piece of apparatus is the rotational inertia disk and assembly (Fig. 3). The disk is mounted on a horizontal axis in precision pivot bearings, so as to turn with negligible friction. It is made in two cylindrical steps. of simple geometric form, so that the rotational inertia of each part can be readily calculated. Upon the plane face of the large disk there is fastened with bits of Scotch tape a sheet of coated paper, with polar-coordinate rulings in degrees. A spark point, mounted on a simple slide with insulating supports, is arranged to move across the face of the disk as it rotates. High-potential sparks, passing at regular intervals from the point to the disk, puncture the paper, and the heat thus developed melts a bit of the paraffin coating and causes a clearly recognizable spot. The location of these marks upon the printed scale gives the angles traversed during successive equal time intervals. The wheel bearings are supported by a frame so designed as to permit the thread which accelerates the disk to clear the edge of the table. The accelerating mass is attached by a light silk cord wound around the rim of the disk and fastened to a pin on its periphery.

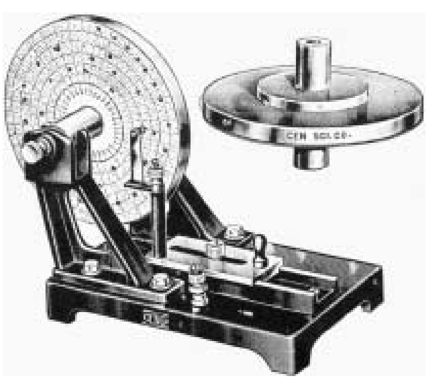

Fig. 3. Rotational Inertia Apparatus. The wheel is shown covered with the coated-paper chart with polar-coordinate rulings.

The sparking mechanism, Fig. 4, consists essentially of a buzzer type of mechanical interrupter vibrating synchronously with the 60-cycle alternating current power supply and thus interrupting the current at intervals of exactly 1/60 second. This interrupted current, passing through the primary of a step-up transformer, induces a high voltage in the secondary of the transformer, the design of the electrical circuit being such that a powerful spark occurs only at the instant of peak voltage. The usual modern ac power supply is so well regulated in frequency that it may be assumed to be exactly 60 cycles per second.

As additional auxiliary apparatus there are needed a vernier caliper, a set of slotted iron masses of 1gm to 200gm, a 50gm weight holder, a spool of silk thread, several coated charts, a roll of Scotch tape, scales capable of weighing the disk, a half-meter stick with caliper jaws, 6-volt storage battery, a double-pole, double-throw switch, stop watch or clock, two C-clamps, scissors, an outside caliper, and a low-friction pulley mounted on a long vertical support rod.

Fig.4. Spark Timer.

PROCEDURE:

Experimental: 1. For a preliminary trial fasten a used sensitized chart to the disk by means of a few pieces of Scotch tape touching the edge of the chart and bending over the periphery of the wheel. Replace the disk on its supports and tighten the knurled screw: Wrap the silk cord several times around the outside rim, fastening one end to the pin provided for that purpose and the other end to the accelerating mass. A fall of the latter of one and one half to two meters is desirable. This may be obtained by passing the string over a high pulley, as shown in Fig. 5.

2. Determine the' frictional torque by attaching a small initial mass m_f to the string. Give the disk a *small* initial speed and continue to adjust m_f until the disk continues to rotate *uniformly* at this speed. The torque thus applied is just equal to the frictional torque, $L_f = m_f gr$.

3. Set the slider so the spark point is near the rim of the wheel. The spark point should be adjusted so that it is slightly less than 1mm from the chart. Attach an accelerating mass of about 200gm in addition to m_f. Call the total mass *m* (*m* = 200 gm + m_f). Release the mass without imparting any speed to it and, as it descends, slide the spark point slowly and uniformly toward the axis. Adjust the speed of the slider so that the spark dots are clearly separated for the successive rotations and the point reaches the end of its motion at nearly the same time as the end of the fall.

4. After the method of moving the spark point has been mastered, connect the spark-timing device to the binding posts of the moment-of-inertia apparatus, turn on the switch, release the falling mass, and gradually move the spark point toward the axis of the disk. Just before the accelerating mass reaches the floor, turn off the spark timer and stop the disk. Examine the trace to see that there are no overlapping

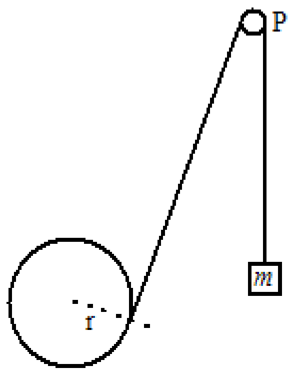

Fig. 5. Arrangement for attaching the accelerating mass *m* over a pulley P to the disk.

rings and few missing points.
If the practice trace appears satisfactory, attach afresh sheet of coated paper and prepare a record trace. Remove the chart and examine the trace. If a few spots are missing the traces, can be used, but if numerous points are missing anew record must be made.

5. Because the time interval between the sparks is very short (1/60 sec) there are more spots than are needed. A larger interval, such as 1/10 sec, is more convenient. Lay the coated paper with the final trace flat on a table under good light. Start with some point near the beginning of the record and mark every *sixth* spot by encircling n by means of a sharp pencil. Number the encircled spots 0, 1, 2, 3, etc. Tabulate the data as shown in the accompanying table.

TABLE I

Spot Number	Angular position, in degrees	Angular distance, in degrees	Average angular speed, in degrees/sec

6. Read the angular position of each consecutive encircled spot. Record these values in degrees in the second column of the table. For each complete revolution add 360˚.

7. Measure the various dimensions of the disk and axle as shown in Fig. 6. Measure and record the mass of the disk, if it is not given with the apparatus.

COMPUTATIONS AND ANALYSIS: 1. Subtract each item in the tabulated column of positions in Table I from the next successive item. This difference is the *angular distance* passed over in each successive time interval. Tabulate these values in the third column. Divide each of these angular distances by the time interval to obtain the *average angular speed* during the interval. Record these values in the fourth column. This average angular speed during the interval is also the *instantaneous* angular speed at the midpoint of the interval.

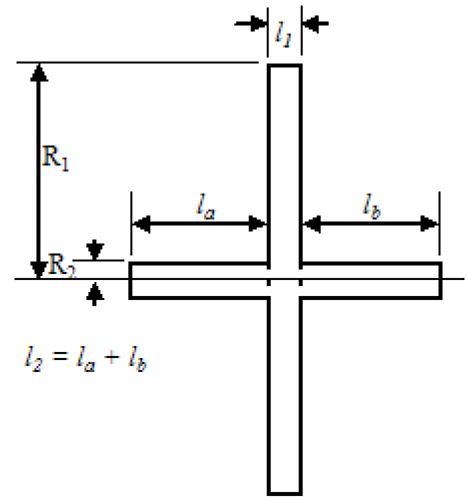

Fig. 6. Dimensions of the two-step rotational inertia wheel.

2. Plot a curve showing the relationship between instantaneous angular speed and time. For this purpose use the data of the fourth column plotted at the *midpoints* of the successive intervals. Carefully interpret the shape, slope, and intercepts of this curve. Compute the slope of the curve, which is the angular acceleration of the disk. Reduce the angular acceleration to radians per second per second.

3. By the use of Eq. (11) compute the moment of inertia of the disk. In this equation the radius *r* is the radius of the part about which the thread was wrapped.

4. *Computed Value of I*: The disk is made of two steps of different radii. The total rotational inertia *I* is the sum of the separate rotational inertias of the two cylinders. In symbols

$$I = \frac{1}{2}\left(M_1 R_1^{\;2} + M_2 R_2^{\;2}\right) \tag{12}$$

where M_1 = mass of large disk
M_2 = mass of shafts
R_1 = radius of large disk
R_2 = radius of shafts

The masses of the two parts may be stated on the apparatus. If not they may be calculated by multiplying the density and respective volumes of the parts. The density may be obtained by dividing the total mass by the total volume.

5. Note the percentage difference between the calculated value of *I* from Eq. (12) and the experimental value from Eq. (11).

Optional Analysis: 1. Plot a curve showing the relation of the total angular distance traversed to the times required to move those distances. Discuss the shape of the curve.

2. By choosing corresponding values of total angular distances traversed and the appropriate average angular velocities, plot an angular velocity vs. angular distance curve. Explain its shape.

3. Select some point on the curve of angular distance vs. time and draw a tangent to the curve. From the slope of the

tangent determine the angular velocity at that instant and compare it with the computed value.

QUESTIONS: 1. The first spark point does not occur when the angular velocity is zero. What effect does this have upon the observed value of α? Explain.

2. If data were taken for angular distances traversed after the accelerating force had been removed and the corresponding angular velocities plotted against time, what sort of curve would be expected? Why?

3. From the units of torque and angular acceleration, show that the absolute metric unit of rotational inertia is the gram-centimeter^2.

4. Explain how, by a simple experiment involving rotational inertia, it would be possible to determine which of two identical-appearing eggs was hard cooked and which was uncooked.

5. Derive a symbolic expression for the linear acceleration of a sphere which starts from rest and rolls down a plane inclined an angle θ to the horizontal. (*Answer*: $a = 5/7 g \sin\theta$. The value of I for a sphere is $I = 2/5mr^2$.)

6. What portion of the total kinetic energy of a rolling solid disk is energy of translation, and what portion is energy of rotation? What are these portions for a thin ring?

7. One end of a string is attached to a 200gm mass. The other end is wrapped around a disk 20.0cm in diameter and of mass 3.00kg. The disk is free to rotate with negligible friction about its horizontal axis. The 200gm mass starts from rest and descends 1.00m. Find the following: (a) the loss of potential energy of the system; (b) the angular acceleration of the disk; (c) the gain in kinetic energy of the descending mass; (d) the tension in the string.

8. A flywheel is made of a uniform cylindrical disk of diameter 60cm and mass 7.0kg. A *constant* force of 2.45×10^5 dynes is applied tangentially at the rim of the disk. What is the angular speed of the disk 0.50 min after it is started from rest?

9. An object of mass 100gm is attached to a cord passing over a disk 20cm in diameter and of mass 3.0kg, free to rotate about a horizontal axis. What is the speed of the 100gm object 0.60 sec after it starts from rest?