Selective Experiments In Physics

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ARCHIMEDES' PRINCIPLE

OBJECT: To study Archimedes' Principle and to apply this principle to determine the density of solids and liquids.

METHOD: A body is weighed in air and then weighed when submerged in a liquid. The apparent loss of weight is, by Archimedes' Principle, equal to the weight of the liquid displaced by the body. From these measurements, the density and specific gravity of the solids and liquids used in the experiment may be determined.

THEORY: The fact that an object immersed in a *fluid,* liquid or gas, should be "buoyed up.' by a force equal to the weight of the fluid it displaces was deduced by Archimedes (287-212 BC). This principle, called Archimedes' Principle. applies to any object in any fluid, for example, a submarine in water or a dirigible in air. If a free body remains at rest when totally immersed in a fluid, there must be no resultant force acting and, hence, the weight of the body must be equal to the weight of the displaced fluid. The body will sink if its weight exceeds that of the displaced fluid and. conversely, rise if lighter. Thus a block of cork released in water, bobs up to the surface and floats like a ship. A ship will adjust to a depth in water for which its weight just equals the weight of water (and air) displaced. The contribution of the air to the buoyant force of a ship is a negligible amount and, therefore, disregarded. The dirigible is, however, totally supported by the buoyant force supplied by air displacement.

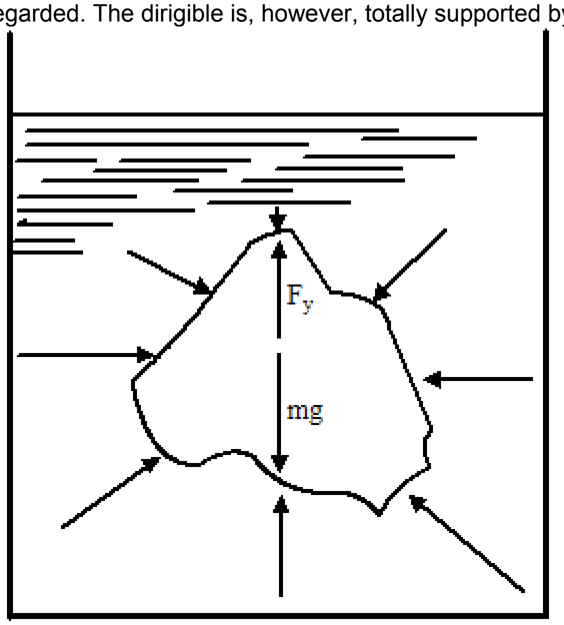

Fig. 1 - Archimedes' Principle - $F_y = mg$

Proof of Archimedes' Principle: Let the irregular outline of Fig. 1 contain any desired portion of a fluid at rest. The arrows are representative forces acting against the bounding surface. Each force is perpendicular to its element of surface and the resulting pressure has a magnitude dependent on the depth of the fluid at that point.

Since the fluid is at rest there is no unbalanced force in any direction. The components of the forces pushing towards the right on the bounding surface must balance those pushing towards the left. The vertical components upward on the surface must support the downward forces on the surface plus the weight of the designated portion of fluid. That is. the resultant upward force F_y must equal the weight *mg* of the fluid contained inside this surface.

When this portion of fluid is replaced by a solid body of exactly the same shape, the pressure at every point is exactly the same as before. Hence, the fluid must exert an upward or buoyant force on the object immersed in it, a force which is again just equal to the weight of the fluid displaced by the object. If the buoyant force is greater than the weight of the object, the object is lifted to the surface. If the weight of the object is greater than the buoyant force, the unbalanced downward force causes the object to sink like a rock in water.

Application of Archimedes' Principle: Archimedes' Principle is experimentally applied to obtain the density *p* of a substance, that is, its mass per unit volume,

$$p = \frac{m}{V} \tag{1}$$

Where *m* is the mass and *V* is the volume of the object.

A body of weight $w = mg$ has an apparent loss of weight when immersed in a fluid. This loss is equal to the weight of the volume of fluid displaced by the object or $w_f = m_f g$. Hence, the apparent weight of the object $m'g$ submerged in the fluid is given by

$$m'g = mg - m_f g \tag{2}$$

Since $m = pV$ (see Eq. 1) one may replace m_f of Eq. (2) with $p_f V$ where p_f is the density of the fluid and V the volume displaced. But the volume of fluid displaced is also the volume of the submerged object, $V = m/p$. Thus, $m_f = p_f (m/p)$ and Eq. (2) becomes,

$$m'g = mg - p_f\left(\frac{m}{p}\right)g$$

or

$$w' = w - \left(\frac{p_f}{p}\right)w$$

and

$$p = \left(\frac{w}{w - w'}\right)p_f \qquad (3)$$

Figure 2 (a) illustrates this method of experimentation.

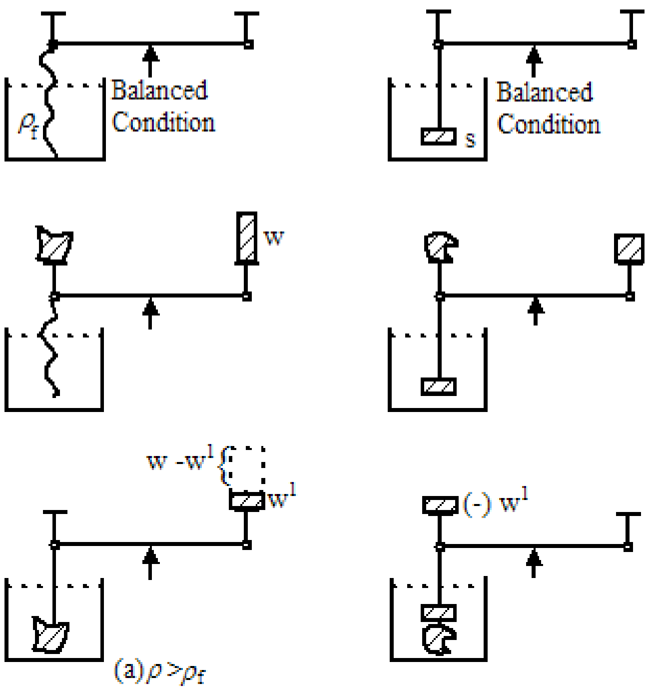

Fig. 2 - Measurements of density using Archimedes' Principle

$$\rho = \frac{w}{w - w^1}\rho_f$$

If the density of the body to be measured is less than the density of the liquid, that is, $p \langle p_f$, it is necessary to fasten a sinker to the body so that the two together will sink in the liquid. If this sinker (see *s* of Fig. 2 (b)) is made a part of the weighing apparatus for all the readings, the value of *p* again may be computed from the two readings required for Eq. (3). Note that the sign of *w*' is now negative. Equation 3 states that the

$$\text{density of object} = \frac{\text{weight of object}}{\text{loss of weight when submerged in fluid}} \times \text{density of the fluid}$$

When the fluid used is water, the portion of this equation within the brackets may be rewritten as

$$sp.gr. = \frac{\text{weight of object}}{\text{weight of equal volume of water}}$$

where *sp. gr.* is called the specific gravity of the substance. Mercury has a specific gravity of 13.6; this means that a given volume of mercury is 13.6 times heavier than the *same volume of water*. To obtain the density of mercury, its specific gravity value must be multiplied by the density of water, i.e., 1gm/cm^3 or 62.4lb/ft^3. Thus, the density of mercury is 13.6gm/cm^3 or 849lb/ft^3. Note that specific gravity is a ratio only and has no units. Density has the units of mass/volume. In the British system the units pounds-per-cubic-foot give a "weight density".

This experiment deals only with solids and liquids which have a very high density compared to air. Thus, the "weight of the object" is accepted as approximately its weight in air.

The relative densities of liquids may be obtained by finding the weights of a given dense object in the various liquids. Suppose that the apparent loss of weight of the object immersed in a liquid of a density p_{f1} is m_1g and when it is immersed in a liquid of density p_{s2} the apparent loss of weight is m_2g. By Archimedes' Principle, $m_1g = Vp_{f1}g$ and $m_2g = Vp_{f2}g$ where *V* is the volume of the object. Hence,

$$V = \frac{m_1}{p_{f_1}} = \frac{m_2}{p_{f_2}}$$

or

$$p_{f_1} = \frac{m_1}{m_2}p_{f_2} \qquad (4)$$

The ratio m_1/m_2 may be measured directly from the corresponding weights. The value of p_{f1} may be computed, provided p_{f2} is known. It is common practice to use water as the comparison liquid since its density (see Table I) is very accurately known.

TABLE I
Density of Water

Temperature °C	Density grams/cm^3	Density pounds/ft^3
0	0.99987	
2	0.99997	62.43
3.98	1.00000	
6	0.99997	
10	0.99973	
15	0.99913	
20	0.99823	
22	0.99780	62.32
24	0.99732	
26	0.99681	

APPARATUS: Trip scale on a supporting stand, Fig. 3; set of masses; overflow can; cylinder or block of wood having a specific gravity of less than 1: irregular metal object (Fig. 4); vernier caliper; liquids of different densities. A dense solution of salt water may be used.

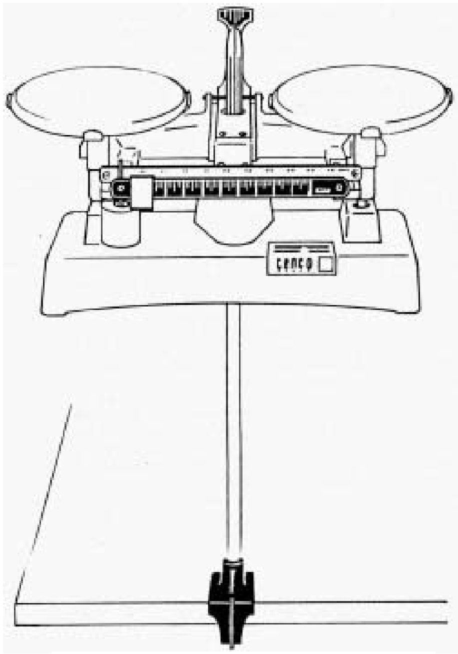

Fig. 3 – Trip scale on supporting stand

Fig. 4 – Overflow can (left) and specific gravity specimen (right)

PROCEDURE:

1. Study of Archimedes' Principle:

A. Floating object. Fill the overflow can with water. Catch the resulting overflow when an object which floats in water is placed in the can. Measure the weight of the overflow water. Weigh the object and compare its weight with that of the overflow. Draw a conclusion.

Note: See that no air bubbles adhere to the objects used in these experiments. The objects must be dry when weighed in air. Some of the data obtained here will be used later.

B. Dense metal object. Fill the overflow can with water. Catch the resulting overflow of water when the dense metal object is immersed in it. Weigh the overflow water.

Weigh the object in air. Next weigh it in the water by attaching a light string between the subject and the underside of one pan of the trip scales (see Fig. 2(a)). Compare the apparent loss of weight of the object with the weight of the overflow water. Draw a conclusion.

II. Density and Specific Gravity of Solids:

A. Determine the volume of the floating object used in part 1 (A) from (a) measurements of its dimensions and (b) the weight of the overflow for complete immersion in water.

Compute the density of this object by using Eq. (1). Express the density in both metric and British units.

What is the specific gravity of this object?

B. Compute the density of the metal object used in part I (B). Use Eq. (3).

C. Obtain the density of the object measured in I (A) using the method shown in Fig. 2 (b). The metal object may serve as the sinker *s*.

III. Density of a Liquid: In part I (B) the loss of weight of the metal object in water was measured. Replace the water with a liquid of different density and find the buoyant force on the object in this liquid.

What is the density of the second liquid?

QUESTIONS: 1. Why is it important that no air bubbles adhere to the objects during measurements? See Note I (A).

2. The specific gravity of gold is 19.3. What is the mass in grams of a cubic centimeter of gold? What is the weight in pounds of a cubic foot of gold?

3. A block of metal having a density of 9.00gm/cm^3 has an apparent weight of 180gm in water and 135gm when submerged in a liquid. What is the density of the liquid?

4. A block of wood and a block of lead of the same mass are weighed carefully on a trip scale using brass weights. Are the weights of these blocks the same? Explain.

5. A submarine floats at rest 50ft deep in water. Without using the propeller how can the captain make it surface? Make it sink?

6. A rock has a specific gravity of 1.30. A man can lift 120 pounds. How many cubic feet of this rock can he lift in air? In water?

7. Oil having a density of 0.80gm/cm^3 floats on water of density 1.0gm/cm^3. A solid object which has a specific gravity of 0.90 is dropped into the container. Locate its exact position of rest.