CENCO Physics Selective Experiments In Physics

• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

COMPOSITION AND RESOLUTION OF CONCURRENT FORCES BY VECTOR METHODS

OBJECT: To study the composition and resolution of concurrent forces as examples of vector quantities.

METHOD: Concurrent forces acting on a body are used as examples of vector quantities. These forces are represented by vectors. The resultant and equilibrant of several sets of such known forces are determined by both graphical and analytical methods. These results are tested on a force table as a check on the first condition for the equilibrium of a rigid body.

THEORY: Measurable quantities may be classified as either (1) scalar quantities or (2) vector quantities. A *scalar quantity* has magnitude only, but a *vector quantity* has both magnitude and direction. For example, since to specify completely the velocity of a body it is necessary to state not only how fast it is traveling but in what direction it is going, velocity is a vector quantity. However, the mass of a body is completely specified by a magnitude, and hence mass is a scalar quantity. Since the weight of a body is the force with which it is attracted by the earth, weight has a downward direction and thus is a vector quantity. Since weight and mass are different physical concepts, they should not be measured in the same units. The gram is a unit of mass. The force with which the earth attracts a one-gram mass at a standard location sometimes is called a "gram-weight” of force. *

*Since weight is proportional to mass in any given locality, this experiment is not affected by the slight variations consequent to laboratory conditions

In order to add scalar quantities, one has merely to make the algebraic addition. When one wishes to add two vector quantities, the process is more difficult because their directions must be considered. The *vector sum* of two vector quantities is the single vector quantity that would produce the same result as the original pair.
The addition of vector quantities is greatly simplified by representing the vector quantity graphically. A *vector* is the line segment whose length represents the magnitude of a vector quantity and whose direction is that of the vector quantity. The sense along the line is indicated by an arrow. For example, a force of 100lb. acting at an angle of 30° above the horizontal may be represented by the line *OA*. Fig. 1, which is 5 units long and has the correct direction. Each unit of length thus represents 20lb.
When vectors do not have the same line of action, their vector sum is not their algebraic sum but a geometric sum. This geometric sum may be determined by either graphical or analytical methods. Graphical methods are simple and direct but are limited in precision to that obtainable by drawing instruments. Analytical methods have no such inherent limitations. In this experiment both graphical and analytical methods will he applied to forces as examples of vector quantities, *but the same methods apply to all vector quantities.*
The *vector sum*, or *resultant*, of a set of forces is the single force that will have the same effect, insofar as motion is concerned, as the joint action of the several forces.

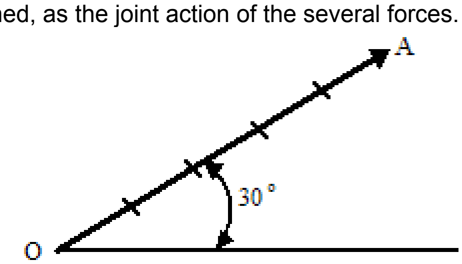

Fig. 1. Representation of a vector by a vector.

Vector Summation by Graphical Methods: As an example of vector addition let us consider the case of two forces acting on a body in such a direction that the forces are *concurrent*, that is their lines of action, if projected would intersect at a point. The vectors *OA* and *OB* representing two such forces are shown in Fig. 2. Their vector sum or resultant **R**, is found by constructing a parallelogram having the two vectors as sides and drawing the *concurrent* diagonal, as shown in Fig. 3. This diagonal vector **R** represents in magnitude and direction the single force that is

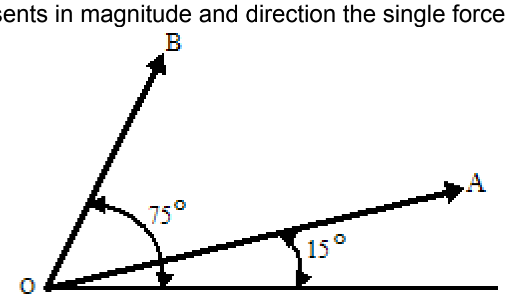

Fig. 2. Two concurrent forces.

equivalent to the origina1 pair, that is their vector sum. When the resultant of more than two vectors is to be obtained graphically a *polygon method* is used. This is illustrated in Fig. 4. The vector **A** is first constructed by the use of a chosen scale and reference direction. Then, from the head of **A**, the vector **B** is drawn. It is clear that the vector **M** is the resultant of vectors **A** and **B**, since **M** would be the concurrent diagonal of a parallelogram if such a parallelogram had been drawn, as was done in Fig. 3. Similarly, it follows that the vector **R** is the resultant of **M** and **C** or of **A**, **B,** and **C**. When the resultant of several forces is required this method is simpler than the parallelogram method. It should be noted that when the parallelogram method is used, the arrows, with their tails together, all radiate from a common point. But in the polygon method the tail of the second arrow coincides with the head of the first, etc.

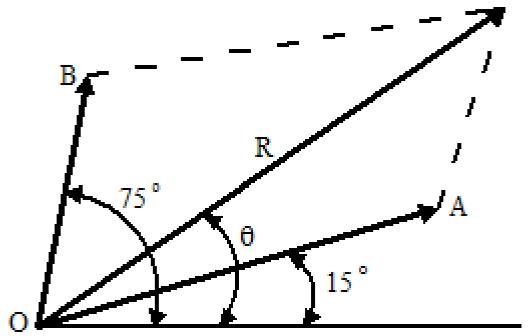

Fig. 3. Parallelogram of forces.

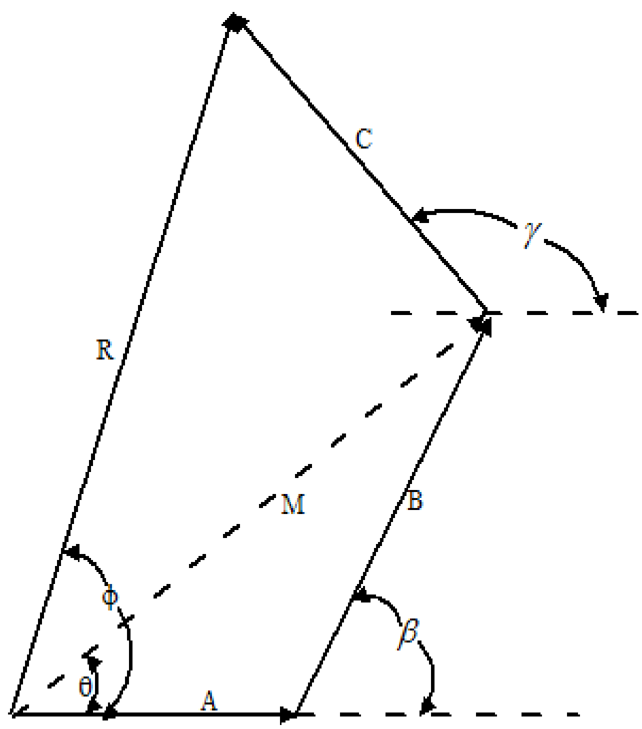

Fig. 4. Vector polygon.

Summation of Vectors by Analytical Methods: The resultant of two vectors may be determined analytically by the use of the trigonometric laws of sines and cosines. Consider the vectors **A** and **B** in Fig. 5. The magnitude of the resultant **R** can be obtained by the application of the law of cosines:

$$R^2 = A^2 + B^2 + 2AB\cos\beta \qquad (1)$$

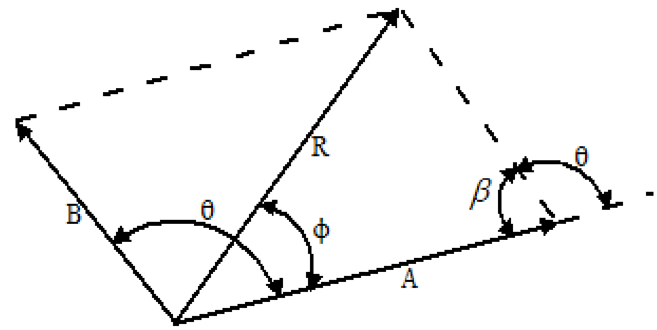

Fig. 5. Law of sine and cosines.

The direction of **R** can then be obtained from the law of sines:

$$\frac{Sin\phi}{Sin\beta} = \frac{B}{R}$$

Since sinβ = sinθ,

$$Sin\phi = \frac{B}{R}\sin\theta \qquad (2)$$

Components of Vectors: Any single force may be replaced by two or more forces whose joint action will produce the same effect as the single force. These various forces are said to be *components* of the single force. The most useful set of components is usually a pair at right angles to each other, as shown in Fig. 6.

The force **B** is the resultant of Forces $\mathbf{B_x}$ and $\mathbf{B_y}$. Therefore conditions are unchanged by replacing the single force **B** by forces $\mathbf{B_x}$ and $\mathbf{B_y}$, called their *X*- and *Y*- components. It is obvious from Fig. 6 that $\mathbf{B_x}$ = B cosβ and $\mathbf{B_y}$ = **B** sinβ.

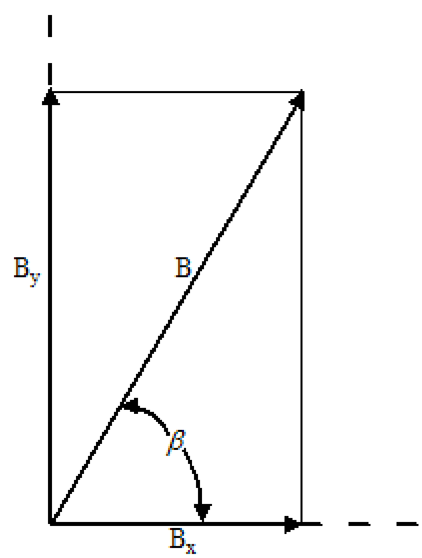

Fig. 6. Components of a vector.

Component Method for Addition of Vectors: Fig. 7 illustrates the component method of computing the resultant of **A**, **B**, and **C**. The *X*- axis is so chosen that it coincides with the vector **A**, and the vectors **B** and **C** are resolved into

Fig. 7. Force table.

X- and *Y*-components. The three forces **A**, **B**, and **C** have been replaced by five forces (**A** has no Y- component). The slim of the component along either axis may be computed by algebraic addition. Calling the sum of the *X*-components F_x and the sum of the *Y*-components F_y, it follows that the resultant **R** is given by the equation

$$R^2 = (F_x)^2 + (F_y)^2 \tag{3}$$

and that the angle ϕ- the angle that **R** makes with the *X*-axis may be determined from the equation

$$\tan\phi = \frac{F_y}{F_x} \tag{4}$$

Equilibrium: Many problems that concern the physicist and engineer involve several forces acting on a body under circumstances in which they produce no *change* in the motion of the body. This condition is referred to as *equilibrium*. The body does not necessarily have to be at rest, but its motion must retain the same velocity; hence both magnitude and direction of motion are unchanged.

First Condition for Equilibrium: Insofar as linear motion is concerned, a body is in equilibrium if there is no resultant force acting upon it, that is if the vector sum of all the forces is zero. This statement is called the *first condition for equilibrium*. This condition is satisfied if the vector polygon representing all the external forces acting on the body is a closed figure. Analytically this condition is satisfied if each set of rectangular components of the forces separately adds to zero, or

$$R_x = \sum F_x = 0 \tag{5}$$

$$R_y = \sum F_y = 0 \tag{6}$$

Equilibrant of a Set of Forces: This is defined as that single force that must be applied to keep a body in equilibrium when it is under the action of other forces. This *equilibrant* (sometimes called anti-resultant) must be equal in magnitude and opposite in direction to the resultant of the applied forces. In a vector polygon the equilibrant would be represented by the vector that closes the polygon. In Fig. 4 the equilibrant for the forces **A**, **B**, and **C** would be the line **R** but with the arrowhead directed *toward* the origin.

APPARATUS: Force table, Fig, 7; scale pans; set of slotted masses; level tester; ruler; protractor, graph paper.
The force table has a circular top, calibrated in degrees, and mounted on a tripod base, equipped with leveling screws. The body whose equilibrium is under study is the ring at the center of the table. The central pin holds this ring in position when the weights are unbalanced. The forces acting on this ring are the tensions in the cords. If the friction in the pulleys is negligible the tensions in the cords are equal to the downward pull of gravity on the suspended masses. Each pulley clamp has an index by means of which the direction of the corresponding force may be read on the circular scale.

PROCEDURE: Each student will select or be assigned one of the problems in Table I (or a similar problem) by the instructor.

1. Using the parallelogram method, determine graphically the resultant and equilibrant of **A** and **B**. Choose a scale such that the finished vector diagram will almost fill the sheet of paper. Use a sharp pencil and construct as accurately as the instruments will permit. Check the results on the force table.

Set up the two given forces by adding to the mass of the scale pans the appropriate slotted masses. Apply the equilibrant force that was obtained from the parallelogram figure. Be sure that the strings are lined up on the ring so that their lines of action will intersect at the center pin. Try displacing the ring and noting its return. If it does not return to the center adjust the equilibrant force to produce equilibrium. Record the final value of this force and compare it with the value found from the vector diagram. Ask the instructor to check your final results.

2. Calculate the resultant and equilibrant of the two given forces used in step 1 by the use of the laws of sines and cosines. Check the result of these calculations on the force table and also compare them with the values determined by the graphical parallelogram method.
3. Select or obtain from the instructor a set of three forces and determine graphically their resultant and equilibrant by using the vector polygon method. Test your results on the force table.
4. With the same three given forces used in Step 3 calculate their resultant and equilibrant by mean is of the analytical component method. Compare this result with the values determined from the vector polygon method.

QUESTIONS: 1. The forces in this experiment act on a ring but are said to be concurrent. Explain. If the cords were attached rigidly to the ring would the forces necessarily be concurrent?

2. Indicate whether each of the following is a vector or scalar quantity: speed, velocity, mass, weight, work, torque, volume.
3. A hammock is supported by two hooks at the same level. A man is seated in this hammock. Under what

conditions will the pull on each hook be equal to the man's weight?

4. A body, weight **W**, is attached by a string, length *l,* to a hook on a vertical wall. A horizontal force **F** acting on the body holds it at a distance d from the wall. Derive the equation which gives the force **F** in terms of **W**, *l*, and *d*.

5. If two forces **A** and **B** are in line the resultant is $A \pm B$. Show how this follows from Eq. (1).

6. Mention a few familiar examples from everyday life of concurrent forces in equilibrium.

7. Since the scale pans used with the force table have equal weights would it be safe to ignore these weights in this experiment? Explain.

TABLE I

	A		B		C	
Problem No.	Force (grams weight)	Angle (degrees)	Force (grams weight)	Angle (degree)	Force (gram weight)	Angle (degrees)
1	150	0	110	70	250	135
2	200	0	100	55	200	135
3	200	0	100	41	150	132
4	200	0	200	97	150	138
5	150	0	200	79	150	154
6	100	0	200	71	160	144