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# ENERGY TRANSFORMATIONS IN MULTIPLE COLLISIONS (LINEAR AIR TRACK)

**OBJECT**: To study energy transformations in multiple collisions of objects on a linear air track and to relate the rate of energy change to a Die-away Curve.

**METHOD**: An object floating on a linear air track is made to collide with several different materials. Upon impact the object rebounds and recollides several times, reaching a particular height for each rebound. Heights of successive rebounds are measured, and the coefficients of restitution are computed. Next, Die-away Curves are drawn from these data to show the rate of energy transformation. The curves then are related to other decay processes.

**THEORY**: A collision is characterized by a relatively large force acting on each colliding body for a short time. When a moving automobile crashes into a concrete bridge abutment, its recoil speed is less than the striking speed. The crushed fenders are evidence that a large force was involved in the collision and that a portion of the kinetic or mechanical energy which the car had before impact was "lost" in doing work to deform the metal. Similarly when a ball is dropped onto a hard surface, it will not rebound to its initial height. Repeated rebounds of the ball will raise its temperature sufficiently to show that there was a transformation of mechanical energy into heat energy.

*Types of collisions.* Even though there is a conservation of total energy in every collision, mechanical energy is always lost (transformed) during collisions of ordinary bodies. Such collisions, called *inelastic collisions*, are the proper study of this experiment.

The fraction of the mechanical energy transformed into non-mechanical energy in a collision varies from 0 to 100%. A steel ball striking a hardened steel block may retain up to 90% of the kinetic energy it had on impact. A wood ball striking the same block retains a much smaller portion.

The single terrestrial exceptions to the inevitable loss, or transformation, of energy during collisions are observed in the case of collisions between certain atomic and subatomic particles. In these collisions there is a conservation of kinetic energy, hence the collisions are *perfectly elastic*. (These particles do not come into physical contact in the usual sense; they interact through electrical and magnetic forces.)

A similar but not identical type of collision is that which is involved in the conduction of sound waves. Since sound can be propagated over very long distances, many, many molecules must be involved in the process of colliding with their neighbors to transmit the sound impulses. If an impulse can be so transmitted through such a large number of molecules, the collisions themselves must approach the condition of near-perfect elasticity.

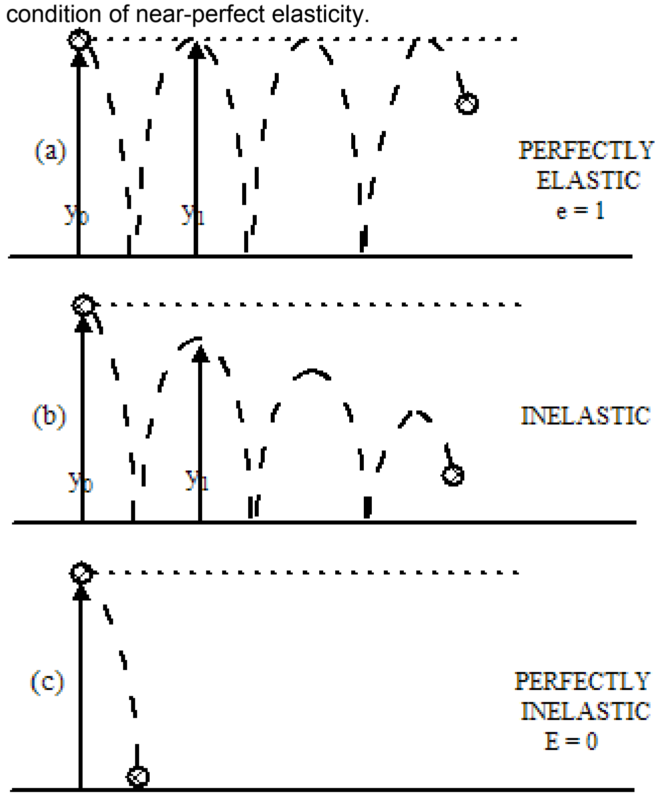


Fig. 1. Types of collisions: (a) perfectly elastic, (b) inelastic, and (c) perfectly inelastic.

At the other extreme, a perfectly inelastic collision requires the complete transformation of mechanical energy; there is a zero rebound velocity after impact. A ball of putty dropped onto a steel plate shows no apparent rebound. The permanent deformation of the putty sphere indicates that the kinetic energy which it possessed on striking was used to rearrange the structure of the sphere. Any head-on collision where the colliding bodies lock together is a perfectly inelastic collision. A bullet captured by the fixed wood block it has penetrated is an example of a perfectly inelastic collision if both block and bullet are motionless after the collision.

The three types of collisions discussed above are represented in Fig. 1, which depicts the rebound of spheres of different elastic characteristics dropped onto a very massive hard surface. Figure 1 (a) shows no loss of kinetic energy; the multiple rebound heights are all equal, hence this is a perfectly elastic collision. Figure 1 (c) shows a perfectly inelastic collision with zero rebound. Figure 1 (b) is a typical inelastic collision in which the height of each successive rebound is less than that of the preceding one, thus indicating that mechanical energy is transformed in each collision.

*Coefficient of restitution.* In *all collisions* the momentum of the system is conserve, that is,

$$mu + MU = mv + MV \qquad (1)$$

where *m* and *M* are the masses of the colliding bodies; and *u, U, v*, and *V* are their respective velocities before and after the collision. See Fig. 2.

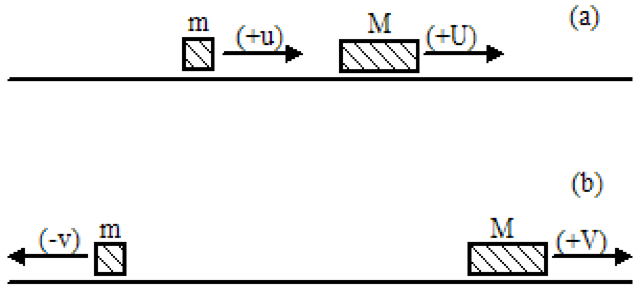


Fig. 2. Conservation of momentum: (a) before collision, (b) after collision.

The kinetic energy may be totally, partially, or not at all conserved in the collision. Expressed in equation form, this statement becomes

$$\frac{1}{2}mu^2 + \frac{1}{2}MU^2 \leq \frac{1}{2}mv^2 + \frac{1}{2}MV \qquad (2)$$

(in which $\leq$ reads equal to or less than). Combining Eq. (2) and Eq. (1) gives

$$v - V \leq -(u - U) \qquad (3)$$

Equation (3) states that the relative velocity of the colliding bodies after collision is equal to or less than the relative velocity before impact.

The last equation is conveniently expressed by the ratio

$$(v - V)/(U - u) = e \qquad (4)$$

where *e*, the constant for a given collision, is called the *coefficient of restitution*. The value of *e* varies from *e* = 1 for a perfectly elastic collision to *e* = 0 for a perfectly inelastic collision.

When the collisions are comparable to the behavior of the rebounding spheres of Fig. 1, where *U* = 0 and *V* = 0, Eq. (4) assumes the simple form

$$v/(-u) = e = \sqrt{y^1/y} \qquad (5)$$

where *y* and $y^1$ are respectively the initial and rebound heights for that impact. Equation 5 follows from the free fall energy consideration, $mgy = 1/2mv^2$ or $v \propto \sqrt{y}$

*Multiple rebounds.* Figure 1 (b) shows the multiple rebounds typical of a spherical object striking a hard fixed surface. The rebound height $y_1$ following the first impact is drawn to be approximately 0.8 of the initial height $y_o$ from which the ball fell. This is expressed by $y_1 = y_o\,(p)$, where (*p*) in this case is 0.8. Making the logical assumption that in succeeding rebounds the ratio of the height of rebounds to the height of drop also will be (*p*), one may write for the second impact

$$y_2 = y_1(p) = (y_o p)(p) = yp^2$$

Similarly $y_3 = y_o p^3$, leading to the general equation

$$y_n = y_o p^n \qquad (6)$$

where *n* is the number of the successive multiple rebound. Thus, to solve for the rebound height after five impacts, accepting an initial height $y_o$ = 90cm and *p* = 0.8, then $y_5$ = 90cm $(0.8)^5$ = 29.5cm. The graph of Fig. 3 shows the computed rebound heights of this bouncing ball for the first fourteen successive impacts. Note that approximately half of the kinetic energy is lost in the first three impacts.

Plotting the amplitude changes of a vibrating steel spring would give a graph of much the same formulas Fig. 3.

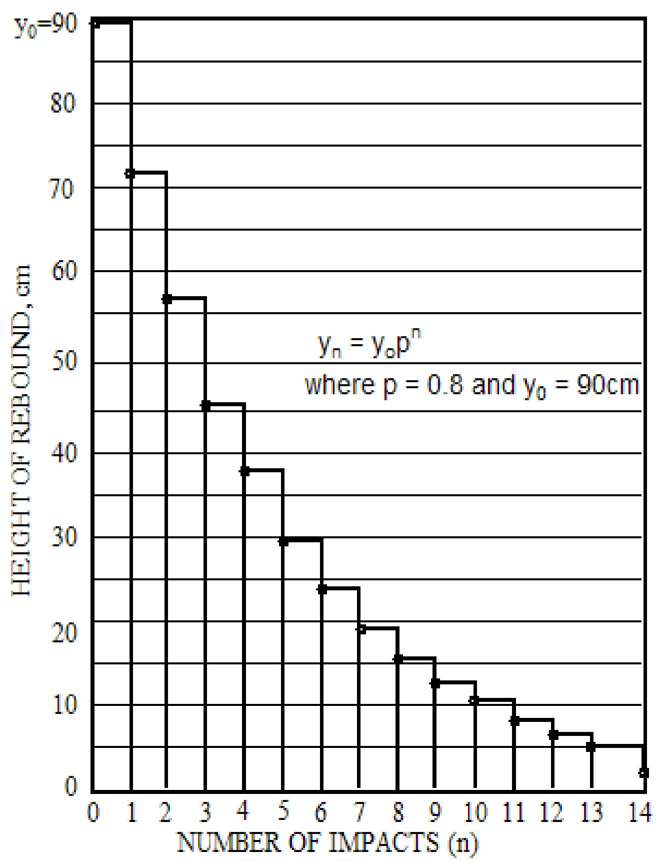


Fig. 3. Height of rebound versus number of impacts of a steel ball on a massive steel block.

The same pattern can be observed in all physical processes in which something is changing at a rate which is proportional to the magnitude of a variable which is changing.

*The Die-away Curve.* Equation 6 is particularly application cases were the independent variable is time, that is, when the transformation or decay proceeds gradually. Two of the many processes characterizing this gradual change are (1) the change with time of the potential of a charged capacitor as it slowly discharges through a very high resistance resistor and (2) the "decay" of a radioactive sample.

The curve representing a gradual decay process is shown in Fig. 4 as a line connecting the several points of Fig. 3. It is called the decay or "*Die-away Curve*." For Fig. 4 assume that the initial ordinate value labeled $N_o$ is 90 milligrams of a radioactive substance. Let the abscissa be time measured in years. Note that this curve, typical of the decay of all radioactive substances, approaches the ordinate zero value at an infinitely slow pace. Thus, no meaningful answer can be given to the question: How long will the radioactive sample last?

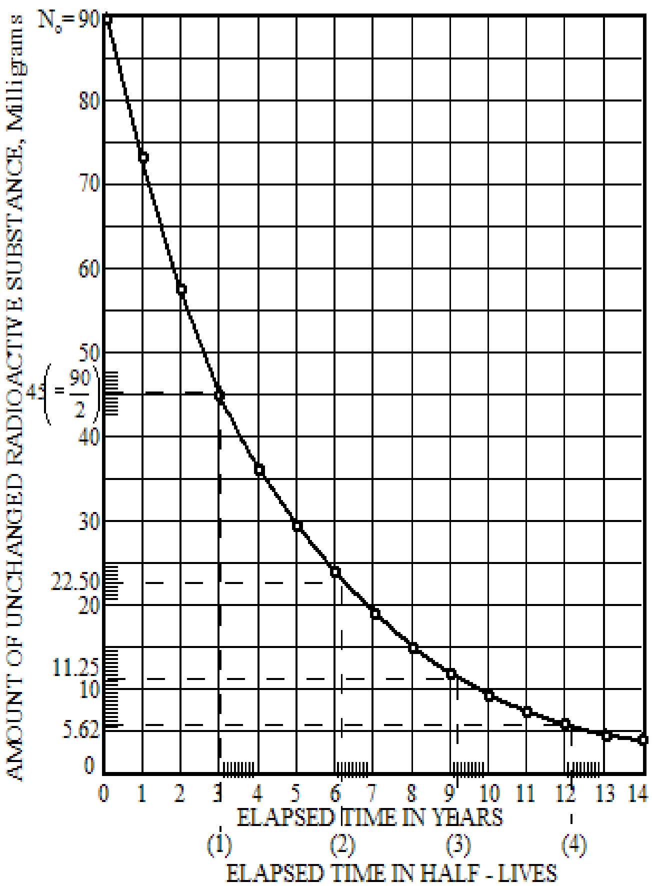


Fig. 4. Typical radioactive decay curve.

*Half-life.* A satisfying answer, however, can be given if the above question is changed to read: In what time span will half the radioactive sample disintegrate? This time is called the "*half-life*" of the substance. Note that in Fig. 4 the initial amount of radioactive material $N_o$ (assumed 90 milligrams) decays to $N_o/2$ (45 milligrams of unchanged material) after an elapsed time of 3.05 years. This, then, is the half-life of that particular radioactive substance. Half of the unchanged material remaining at the end of 3.05 years will, in turn, undergo radioactivity in the next 3.05 years. Note that the abscissa time intervals for successive half-lives are all the same.

*The linear Air-Track.* To apply the theory given, the fall and rebound of the colliding body must occur under a condition of essentially zero-resisting force between impacts. Dropping an object directly onto a massive hard surface results in a succession of rebounds which are too rapid to permit visual reading of successive rebound distances.

It is possible, however, to take these readings on a linear air track, where the resistance offered to the "falling" object (glider) between impacts is essentially zero, and the rebounding object reverses direction at its successive uppermost positions slowly enough to permit the visual reading of these positions.

**APPARATUS**: Linear air track and its source of compressed air, (Fig. 5 and Fig. 6); glider to serve as the bouncing body; rubber band and steel spring, attached to the air track, into which the glider impacts.

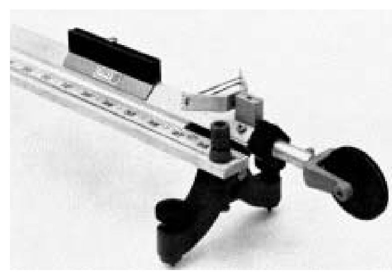

Fig. 5. Basic setup for making collision measurements on the Cenco Linear Air Track.

The linear air track is shown in Fig. 5. Air from the compressed air source rushes out of the many small holes of the air track to support the glider. The glider floats along the slightly elevated track on a cushion of air, being restrained in its motion only by the negligibly small resistance of the surrounding atmosphere.

**PROCEDURE**: Place the glider on the horizontally positioned air track. Apply only sufficient air pressure to make the glider float. Make any transverse adjustments of the track necessary for the glider to move along the track with no metal-to-metal contact.

(1) Place a rubber band on the rebound frame and attach the frame firmly to the air track. Elevate the other end of the track about 1.5cm. Release the glider from the elevated end of the track and note the maximum height of its successive rebounds. If there is any indication that the glider does not move freely, make additional adjustments of the track.

Releasing the glider at the top end of the track, read the rebound distances of the glider on successive rebounds.

These distances are proportional to the vertical height through which the glider falls. It is suggested that one student read the positions and call out the values while his partner records them.

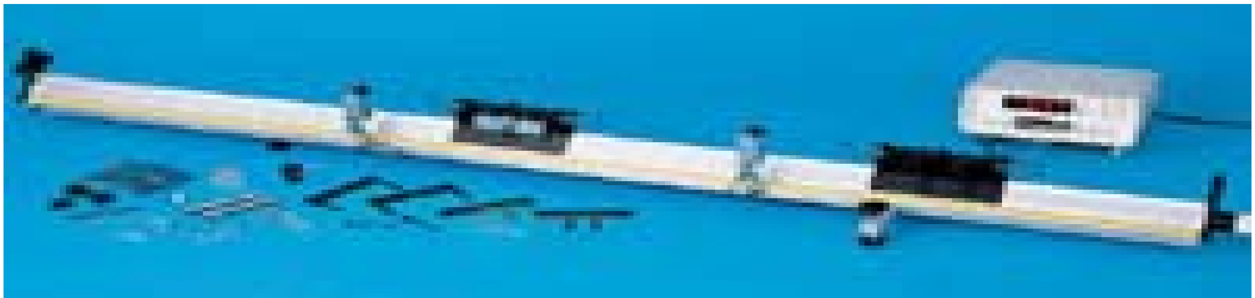

Fig. 6. Cenco Student Air Track and its experimental accessories.

(2) Replace the rubber band with a steel spring and repeat the above.

(3) *Qualitative test*. Substitute a material of your choosing on the rebound frame and note the elasticity on impacts.

Using the data obtained in steps (1) and (2) above proceed as follows for each step:

(a) Construct a rebound graph comparable to that shown in Fig. 3.

(b) Add the associated "Die-away Curve."

(c) Compute the coefficient of elasticity involved in these collisions.

(d) Solve for the percent loss in kinetic energy before and after impact for each of the first four impacts. What conclusions can be drawn from these results?

(e) Assuming that the "Die-away Curve" applies to a test on a radioactive material, and using milligrams and years for the x and y axis respectively shown in Fig. 4, what is the half-life of the substance? How long would it take for this substance to lose 90% of its radioactivity?

**QUESTIONS**: 1. A ball falls from an initial height $h$ and strikes a massive steel block. It rebounds to a height of $h/2$. What is the ratio of the striking velocity of the ball to its rebound velocity?

2. The coefficient of restitution $e$ in a collision is 0.5. What percent of the striking kinetic energy is transformed in the collision?

3. Two masses $m_1 = m_2$ have velocities of approach ($v$) and ($-v$). What is the momentum of this system before impact and after impact for a perfectly elastic collision? For a perfectly inelastic collision?

4. Two automobiles each of mass $m$, traveling at the same speed, one west and the other north, collide and lock together. Is there a total transformation of the kinetic energy in this collision? Explain the answer given.

5. An atom of mass $m$ and velocity $v$ collides with an atom of mass $m$ at rest. In another encounter the atom of mass $m$ and velocity $v$ collides with an atom of mass $2m$ at rest. Show that the striking atom does not retain the same amount of kinetic energy in the two collisions.

6. The intensity of parallel light of wavelength $\lambda$ is reduced to half value after penetrating 100 feet of fog. What is the intensity at a distance of 150 feet from the source?

7. Plotting Fig. 3 for the bouncing ball using height of rebound versus time would give a different curve. Why?

8. If the elevation angle of the air track were doubled, would the value of $e$ obtained in this experiment still be the same? Why or why not?